Syllabus for Ph. D. Eligibility Test (PET) 2016
Mathematics / Applied Mathematics

Section (A): Research Methodology:

Unit I: Logical Aptitude.

Unit II: Numerical Aptitude.

Unit III: Fundamentals of Mathematics.

Unit IV: Fundamentals of Computers.

Unit V: English language skills.

Section (B): Mathematics:

Unit I: Algebra: Binary relation, binary operation, function, group, subgroup and their properties. Lagrange's theorem, Fermat's and Euler's theorem and their consequences.Normal subgroup, quotient group and their properties.Homomorphism and related theorems, Fundamental theorem of group homomorphism, automorphism, conjugacy and G-sets. Permutation groups and related concepts and results.Center, normalizer, commutator of a group, derived group, Cayley's theorem.Normal series, solvable and nilpotent group and their properties, direct products, simplicity of alternating group.Fundamental theorem of finitely generated abelian group, invariants of a finite abelian group, Sylow theorems and applications.

Rings, definition, types, subring, ideal, prime, maximal ideas, nil, nilpotent ideals and their properties.Quotient ring, Homomorphism, isomorphism and related results.UFD, PID, Euclidean domain, polynomial rings and their properties.

Extension fields, irreducible polynomials, algebraic extension and their properties, splitting field, normal extension, multiple roots, finite fields, separable extension.Automorphismgroups, fixed field, fundamental theorem of Galois theory, polynomials solvable by radicals, ruler and compass constructions.

Linear Algebra: Vector spaces, subspaces, linear dependence, independence, basis and dimension of a vector space. quotient space, homomorphism, dual space, inner product space and modules

Rank of a matrix, change of a basis, Linear transformations, algebra of linear transformations, range space, kernel space, rank of a linear transformation.Matrix representation of a linear transformation, dual spaces.Eigen values, eigen vectors, Cayley – Hamilton theorem, Minimal polynomials. Canonical forms, Diagonal form , triangular form, Jordan form

Lattice Theory: Partially ordered sets, lattice as a poset, lattice as algebra, Hasse diagrams, planer and optimal Diagrams, meet and join tables, Homomorphism. Isotone maps, sub lattices, ideals and their characterizations congruence relations, congruence lattices, the homomorphism theorem, product lattices, ideal Lattice, complete lattice and their properties.

Distributive and modular inequalities and identities, complements and pseudo complements Demorgan's identities, Boolean lattice of pseudo complements, meet and join-irreducible elements, characterization theorems and representation theorems Dedikind's modularity criterion, Birkhoff'sdistributivity criterion.

Hereditary subsets, ring of sets, Stone theorems, Nachbin theorem Distributive join-semi lattices and characterization, Distributive lattices with pseudocomplementation.

Join infinite distributive identify, algebraic lattices stone algebra and its characterizations.

Unit II: Real Analysis: Definition and existence of Riemann-Stieltjes integral, Properties of the integral, Integration and Differentiation, The fundamental theorem of calculus, Examples.

Integration of vector valued functions. Rectifiable curve.Examples.Sequences and series of functions.Point wise and uniform convergence.Cauehy criterion for uniform convergence.Weierstrass M-test, uniform convergence and continuity, uniform convergence and Riemann-Stielljes integration.Examples.

Uniform convergence and Differential, The Stone – Weierstrass theorem, Examples.Power series, Abel's and Taylor's theorems, Uniqueness theorem for power series.Examples. Functions of several variables, Linear transformations, Derivatives in an open subset of R^n, Chain rule, Examples

Partial derivatives. Interchange of the order of differentiation, The inverse function theorem, The implicit function theorem Jacobins, Derivatives of higher order, Differentiation of integrals. Examples,

Measure Theory: Measure on the real line.Lebesgue outer measure, measurable sets. Regularity. Measurable functions. Borel and lebesgue measurability.Examples.

Integration of functions of a Real variable.Integration of a simple function.Integration of non-negative functions.The general integral.Integration of series.Examples.

Riemann and Lebesgue Integrals, Differentiation. The four derivates, Functions of bounded variations. Lebesgue's differentiation theorem differentiation and Integration.Examples.Abstract Measure spaces.Measures and outer measures Extension of a measure.Uniqueness of the extension.Completion of a measure spaces.Integration with respective to a measure.Examples.

The L^p spaces. Convex functions. Jensen's inequality. The inequalities of Holder and Minkowski Completeness of L^p (μ) Convergence in measure. Almost uniform convergence.Examples.

Complex Analysis: The field of complex numbers, The complex plane, Rectangular and polar representation of complex numbers; Intrinsic function on the complex field; The Complex plane.

Definition and examples of metric spaces; connectedness; sequence and completeness; compactness; continuity; Uniform convergence.

Power series; The exponential function; Trigonometric and hyperbolic functions; Argument of nonzero complex number; Roots of unity; Branch of logarithm function. Analytic functions; Cauchy Riemann Equations; Harmonic function;

Analytic functions as a mapping; Mobius transformations; linear transformations; The point at infinity; Bilinear transformations, Power series representation of analytic functions; zeros of an analytic function.

The index of a closed curve; Cauchy's theorem and integral formula; Gaursat's Theorem; Singularities: Classification of singularities; Residues; The argument principle. Compactness and convergence in the space of Analytic functions.Spaces of analytic functions; Theweierstrass factorization theorem; factorization of the sine function; The gamma function; The Riemann zeta function.

Harmonic functions, Basic properties of Harmonic functions and comparison with analytic function; Harmonic functions on a dick; Poisson integral formula; positive harmonic functions. Entire functions; Jensen's formula; The Poisson-Jenson formula; The genus and order of an entire function. Hadamard factorization Theorem; Univalent functions; the class S; the class T; Bieberbachconjucture; sub class of s; Analytic continuation: Basic concepts; special functions.

Unit III:Differential Equations: Existence, uniqueness and Continuation of solutions: Method of successive approximations for the initial value problem $y^1=f(x,y)$, $y(x_0)=y_0$, The Lipschitz condition. Peano's existence theorem, maximal and minimal solutions, continuation of solutions.

Existence theorems for system of differential equations: Picard-Lindelof theorem, Peano's existence theorem, Dini's derivatives, differential inequalities.

Integral Inequalities: Gronwall- Reid-Bellman inequality and its generalization, Applications: Ziebures theorem, Peron's criterion, Kamke's uniqueness theorem.

Linear systems: superposition principle, Properties of linear homogeneous system, Theorems on existence of a fundamental system of solutions of first order linear homogeneous system, Abel-Liouville formula.

Adjoint system, Periodic linear system, Floquet's theorem and its consequences, Applications, Inhomogeneous linear systems, applications.

Superposition principles, Lagrange Identity, Green's formula, variation of constants, Liouville substitution, Riccati equations Prufer Transformation. Higher order linear equations.

Maximum Principles and their extensions, Generalized maximum principles, initial value problems, boundary value problems.

Theorems of Strum; Sturm's first comparison theorem, Sturm's separation theorem, strums second comparison theorem.

Sturm-Liouville boundary Value Problems: definition, eigenvalues, eigenfunctions, orthogonality.

Number of zeros, Non oscillatory equations and principal solutions, Non oscillation theorems.

Partial Differential Equations: First order partial differential equation, linear equations of the first order, integral surface passing through a curve, surfaces orthogonal to a given system of surfaces.

Non-linear partial differential equations of the first order, Cauchy's method of characteristics, compatible system of first order equations (condition of compability), Charpit's method.

Special types of first order equations, solutions satisfying given conditions,

a) Integral surface through a curve.
b) Derivation of one complete integral from another.
c) Integral surfaces circumscribing a given surfaces.

Jacobi's method for solving $F(x, y, z, p, q) = 0$.

The origin of second order equations, linear partial differential equations with constant coefficients, intermediate integrals or first integrals, Monge's method of integrating $Rr + Ss + Tt = V$, classification of second order partial differential equation (Canonical form).

Characteristic curves of second order equations, the solution of the hyperbolic equations, separation of variables, the method of integral transform, application of partial differential equation (One dimensional wave equation, one dimensional and two dimensional heat flows)

Mechanics: Mechanics of system of particles, generalized coordinates, Holonomic&nonholonomic system, Scleronomic&Rheonomic system, D' Alemberts's principle and Lagrange's equation of motion, different forms of Lagrange's equation, Generalized potential, conservative fields and its energy equation, Application of Lagrange's formulation.

Functionals, Linear functionals, Fundamental lemma of Calculus of Variations simple variational problems, The variation of functional, the extremum of functional, necessary condition for extreme, Euler's equation, Euler's equation of several variables, invariance of Euler's equation, Motivating problems of calculus of variation, Shortest distance, Minimum surface of revolution, Brachistochrone Problem, Isoperimetric problem, Geodesic.

The fixed end point problem for 'n' unknown functions, variational problems in parametric form, Generalization of Euler's equation to (i) 'n' dependent functions (ii) higher order derivatives. Variational problems with subsidiary conditions. Hamilton's principle, Hamilton's canonical equations, Lagrange's equation from Hamilton's principle Extension of Hamilton's Principle to nonholonomic systems, Application of Hamilton's formulation (Hamiltonian) cyclic coordinates & conservation theorems, routh's procedure, Hamilton's equations from variational principle, The principle of least action. Two-dimensional motion of rigid bodies. The independent coordinates of a rigid body, Orthogonal transformations, Properties of transformation matrix, The Euler angles, Cayle-klein parameters & related quantities, Euler's dynamical equation for the motion of rigid body. Kepler's law of planetary motion.

Unit IV: Topology: Prerequisites: Partially ordered sets, Maximal and minimal elements, cardinality, special cardinals countable and uncountable sets, Axiom of choice continuum hypothesis, principle of inductions metric spaces, definition and Examples, continuous map, open sets properties of open sets, characterizations of continuity.

Definition and examples of topological spaces, closed sets, closure of a set, properties of closure of sets, interior of a set and their properties, frontier of sets and its relationship with closure and interior of sets neighborhood of a point, Neighborhood system, accumulation point and derived set.

Bases and sub bases and related theorems, new spaces from old, Sub spaces, continuous functions, product spaces, weak topologies and related theorems, open and closed maps, projection maps.

Evaluation map and related results Quotient spaces sequences in a topological space, Inadequacy of sequences, first countable spaces.

Directed sets, nets, convergence of nets, cluster point, subnet, ultra net, filter, convergence of filters, ultra filters, fixed and free filters, results on these concepts.

Separation axioms, T_0, T_1, T_2spaces and their properties and characterizations, regular spaces, T_3 spaces, characterizations, Hereditariness of these concepts, completely regular and Tychonoff spaces and their characterizations.

Normal spaces and T_4 spaces, Urysohns lemma, Tietzeextension theorem on normal spaces (without proof), cover, point finite cover, shrinkable cover of topological spaces and their properties, countability properties, second countable spaces, Lindelof spaces and their properties,

Compactness, Definition and examples, characterization of compactness, sequentially and countably compact spaces, locally compact specs and their properties, compactification, one point compectification, Stone-cechcompactification.

Para compactness, local finiteness, Metrizable spaces, Lebesgue covering lemma, Urysohnsmetrization theorem, metrizebility of T_0 spaces, Connected spaces, mutually separated sets, characterizations and properties of connected spaces, components, simple chain, path wise and local connectedness. **Functional Analysis:**

Normed linear spaces. Banach spaces and examples, quotient spaces of a normal linear space and its completeness, equivalent norms.

Bounded linear transformations, Normed linear spaces of bounded linear transformations, Hahn-Banach theorem. Conjugate spaces with examples, natural embedding of a normed linear space in its second dual, reflexive spaces

Open mapping theorem, closed graph theorem, projections on Banach spaces, uniform boundedness theorem and its consequences. Inner product spaces, examples.

Hilbert spaces and its properties. Orthogonal complements, orthonormal sets, Bessel's inequality, complete orthonormal sets and parseval's identity, conjugate space of a Hilbert space, reflexivity of a Hilbert space.

Self adjoint operators, positive, projection, normal and unitary operators and their properties Eigen values and eigen space of an operator on a Hilbert space, spectrum of an operator on a finite dimensional Hilbert space Finite dimensional spectral theorem.

Linear Integral Equations: Definition of Integral Equations and Linear Integral Equations, Types of Linear Integral Equations, Special kinds of Kernels: Separable or degenerate kernel, symmetric kernel, convolution-type kernels, Eigenvalues and eigenfunctions of kernels, Solution of linear integral equations, Verification of solution of linear integral equations.

Conversion of Boundary Value Problem to integral equations, conversion of Initial Value Problems to integral equations, conversion of Fredholm integral equations to Boundary Value Problems, conversion of Volterra integral equations into Initial Value Problems.

Methods of obtaining solution for Fredholm integral equations, Fredholm integral equations with separable kernels, Approximating kernels by separable kernels, Method of successive approximation, Iterated kernel method for Fredholm integral equations, Resolvant kernels and their properties, Methods of solutions for Volterra integral equations, Volterra type kernel, Method of differentiation, Method of successive approximations, Method of iterative kernels, Resolvant kernels and its use to solve Volterra integral equations.

Symmetric kernel, trace of a kernel, Fredholm operator, Fundamental properties of symmetric kernels, Eigenvalues and eigenfunctions of symmetric kernel and their properties, normalized eigenfunctions, Iterated kernel of symmetric kernels and their properties, Truncated kernel of symmetric kernel and necessary and sufficient condition for symmetric kernel to be separable, The Hilbert-Schmidt theorem, Solution of a Symmetric Integral equations.

Integral Transform Methods, Recall of Laplace and Fourier Transforms, Applications to Volterra integral equations with convolution-type kernel, examples, Green's function approach for ordinary differential equations.

Unit V: Fluid Mechanics: Review of vector Analysis, Kinematics: Lagrangian and Eulerian methods (Rathy) Real and ideal fluids, velocity at a point, streamlines, path lines, streak lines, velocity potential, irrotational and rotational motions (Rathy), vorticity and circulation, Local and particle rates of change, The equation of continuity. Acceleration of a Fluid.Conditions at rigid boundary, General analysis of fluid motion. Pressure at a point in a fluid at rest and moving fluid, conditions at a boundary of two inviscid immiscible fluids, Euler's equation of motion, Bernoulli's equation. Steady motion under conservative body forces, Potential Theorems, Axial symmetric flows, some two dimensional flows, Impulsive motion, some aspects of vortex motion, sources, sinks, doublets and their images. Some two dimensional flows: Meaning of two dimensional flow, use of cylindrical polar coordinates, The stream function, The complex potential for two dimensional irrotational, incompressible flow, complex velocity potentials for standard two dimensional flows. Examples, two dimensional image systems, Milne-Thomson circle theorem, applications and extension of circle theorem, the theorem of Blasius, conformal Transformation. Viscous flows, stress components in a real fluid, Relation between Cartesian components of stress, translationl motion of a fluid element, rate of strain quadric and principal stresses, properties of the rate of strain quadric.

Stress Analysis in Fluid Motion, relation between stress and rate of strain, the coefficient of viscosity and laminar flow, the Navier Stock's equations, The energy equation,, Equations in Cartesian, cylindrical or spherical polar coordinates for a viscous incompressible fluid: - Diffusion of velocity and dissipation of energy due to viscosity.

Some Solvable Problems in viscous flow with heat transfer: - Flow between parallel Plates velocity and temperature distribution, steady flow through a tube of uniform circular cross section, Velocity and Temperature Distribution, Distribution, steady flow between concentric rotating cylinders, velocity and temperature distribution, Flow in tubes of arbitrary but uniform cross section, equations for velocity and Temperature in a steady flow, Uniqueness Theorem for the velocity and Temperature , Velocity distribution for tubes having equilateral triangular or elliptic cross section, Velocity distribution for the flow through a tube of rectangular cross section.

Flow between two porous Plates, plane Couett of plane poisseuille flow – velocity and temperature distribution, Flow through a convergent or divergent channel,, Flow of two immiscible fluids between parallel Plates, Flow due to a Plane wall suddenly set in motion or due to an oscillating plane wall.

Flows at small or large Reynolds numbers: Dimensional Analysis Non-dimensional form of the Navier Stokes equations, approximate equations for flows at small or large Reynolds numbers, Flows at small Reynolds number: Theory of Lubrication between two plates, Model of a Paint brush, Stoke's flow past a sphere, drag, Flow through a porous slab.

Flows at large Reynolds number: Derivation of the boundary layer equations, Karnans momentum integral equations.

Numerical Analysis: Solution of algebraic and transcendental equations: Introduction; Bisection method; Iteration methods; based on first and second degree equations iteration methods Newton Raphson method; Secant and Regular falsi methods, Rate of convergence for secant method and Newton Raphson method; General iteration methods.

System of Linear Algebraic equations: Introduction; Linear system of Equations: Direct methods; Gauss Elimination method; Iteration methods; Jacobi iteration method; Gauss seidal iteration method; successive over Relaxation (SOR) method.

Interpolation and approximation: Introduction; Interpolation; Langrange and Newton Interpolation Finite difference operators; Interpolating polynomial using finite difference; Hermite interpolation; piecewise and spleen interpolation.

Numerical differentiation and integration: Introduction; Numerical differentiation and integration based on interpolation; Gauss Lagendre interpolation method; Gauss Hermite integration method, Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule.

Numerical solution of ordinary differential equations: introduction; solutin by Taylor series, picards method of successive approximations, Euler method, Modified Euller method, Range – kutta methods.

Fuzzy Mathematics : Specific objectives: To introduce the theory of fuzzy sets as a measure of uncertainty and a ambiguity. Also to introduce fuzzy and fuzzy logic and different operations on them. From classical (crisp) sets to fuzzy sets; Introduction: crisp sets: An overview; Basic concepts in fuzzy sets; convex fuzzy sets (Theorems and exercises)

Fuzzy sets versus crisp sets: Additional properties of α - cuts; Representation of fuzzy sets; Decomposition Theorems. Operations on Fuzzy sets;Types of operations; Fuzzy complement (Axioms and theorems)

Fuzzy intersections: t- norms; fuzzy unions: t – co norms; Combinations of operations; Aggregation of operations.

Fuzzy Arithmetic: fuzzy numbers; Linguistic Variables; Arithmetic operations on intervals of real numbers; Arithmetic operations on fuzzy numbers.

Fuzzy relations: Introduction; fuzzy Relations; operations on fuzzy relations; α - cuts of a fuzzy relation; composition of fuzzy Relations; fuzzy relation on a domain.

Fuzzy Logic: Introduction; three valued logic; Infinite valued logic; fuzzy proposition and their interpretations in terms of fuzzy sets. Fuzzy rules and their interpretations in terms of fuzzy relations.

Operations research: Linear programming problems, convex sets, feasible solutions, formulation of L.P.P. method for solution of LPP.

Graphical solution of L.P.P.Simplex method; theory and problems.Computational procedure, artificial variables inverse of a matrix using simplex method.

Duality in L.P.P. Concept of duality, properties, dual simplex method, its algorithm.parametric linear programming.

Transportation and assignment problems, various methods.

Game theory two person zero sum games, saddle point mixed strategies, graphical solution, by L.P.P., dominance.

Sequencing, problems with n jobs and two machines, problems with n jobs and two machines, graphical method, n- jobs and m machines.

Dynamic programming, computational procedure, solution of LPP by dynamic programming.

Nonlinear Programming introduction, general nonlinear programming problems, problem of constrained maxima and minima, graphical solution Kuhn-Tucker conditions, Quadratic programming.Integer programming.Replacement problems, Applications to industrial problems.Network scheduling and PERT CPM.

WAVELET ANALYSIS AND APPLICATIONS: Vector Spaces, Bases, Orthonormality, Projections, Orthogonal and Orthonormal Functions, Function Spaces, Orthogonal Basis Functions, Orthonormality, Trigonometric series, Euler's formulae, Fourier integral theorem,

Complex Fourier Series, Complex Exponential Bases, Fourier Transforms: FourierIntegral Theorem, Definition and existence of FourierTransform(F.T.), Inverse F.T., Basic Properties of F.T.: Shifting, Scaling, translation, Modulation, Conjugate, Duality. Riemann-Lebesgue Lemma, F. T. of Dirac's delta function, Convolution and parsevals's relation, Poisson summation formula.Windowed Fourier Transform (STFT), Properties, Continuous Wavelet Transform, Discrete Wavelet Transform, Harr Scaling Functions, Function Spaces, Nested Spaces, Harr Wavelet Function, Orthogonality, Normalization at different Scales, Refinement Relation, Support of a Wavelet System, Daubechies Wavelets.

Designing Orthogonal Wavelet system: Refinement Relation for Orthogonal Wavelet systems, Restrictions on filter Coefficients, Designing Daubechies Orthogonal Wavelet System Coefficients, Design of Coiflet Wavelets, Symlets.

Filter Banks: Signal Decomposition, Relation with filter Banks, Frequency Response, Signal Reconstruction: Synthesis from Coarse Scale to Fine Scale, Unsampling and Filtering, Perfect Matching Filters Computing Initial S_{j+1} Coefficients.

Generating and Plotting of Parametric Wavelets: Orthogonality Conditions and Parameterization, Poly phase Matrix and Recurrence Relation, Pollen-type Parameterizations of Wavelet Bases, Numerical Evaluation of ϕ and ψ by various Methods.

Biorthogonal Wavelets: Biorthogonality in Vector Space, Biorthogonal Wavelet Systems and Signal representation, Biorthogonal Analysis, Biorthogonal Synthesis-From Coarse Scale to Fine Scale, Construction of Biorthogonal Wavelet System,

Wavelet Packet Analysis, Haar wavelet packet. Mathematical Preliminaries for B-splines, B-splines scaling function, orthogonalization of Causal /B-splines scaling function, Anti Causal B-splines, Symmetric splines, Differentiation of B-splines, Fractional Splines, Orthogonal Fractional B-Splines.